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EFFECT OF WALL CONDUCTANCES ON HYDROMAGNETIC CONVECTION OF A RADIATING GAS IN A VERTICAL CHANNEL

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Abstract—An exact solution has been found for the fully-developed convective flow of an electrically conducting fluid between two vertical thermally and the electrically conducting walls, under a transverse magnetic field. It is found that with the increase of electrical conductance of the walls, the velocity, the temperature, the flow rate and the rate of heat transfer decrease and the magnetic field increases. But the velocity, the magnetic field, the flow rate, the rate of heat transfer decrease and the temperature increases with the increase of thermal conductance of the walls.

NOMENCLATURE

$a_1, a_2, \ldots, a_8$, constants defined in (24);
c_1, constant defined in (12);
c_2, dimensionless constant, $c_1 L^3/\nu\alpha$;
c_p, specific heat at constant pressure;
d, thickness of the walls;
$e_{b\lambda}$, Planck-constant;
F, radiation parameter, cL^2/α;
g, gravitational acceleration;
H_0, applied magnetic field;
H_z, induced magnetic field along z-direction;
h, dimensionless magnetic field, $H_z/\sigma\mu_e\alpha c_2 H_0$;
k, thermal conductivity of the fluid;
k_w, thermal conductivity of the walls;
k_λ, absorption coefficient of the wall;
L, half-width of the channel;
M, Hartmann number, $\mu_e H_0 L(\sigma/\rho\nu)^{\frac{1}{2}}$;
N, temperature gradient;
p, pressure;
q_R, radiative heat flux;
Ra, Rayleigh number, $g\beta NL^4/\nu\alpha$;
T, temperature;
T_{w0}, reference temperature;
T^*, temperature of the fluid;
u, velocity, z-direction;
u_1, non-dimensional velocity, $uL/\alpha c_2$;
y, z, rectangular coordinates;
α, thermal diffusivity;
β, coefficient of thermal expansion;
η, non-dimensional coordinate, y/L;
θ, non-dimensional temperature, $-\theta^*/NLc_2$;
μ_e, magnetic permeability;
ν, kinematic coefficient of viscosity;
ρ, density of the fluid;
σ, electrical conductivity of the fluid;
σ_w, electrical conductivity of the walls;
ϕ, wall conductance ratio, $\sigma_w d/\sigma L$;
ψ, thermal conductance ratio, $kd/k_w L$.

INTRODUCTION

THE HYDROMAGNETIC combined free and forced convection flow between two vertical parallel plates under a transverse magnetic field, has been studied in a number of works [1–4]. Yu and Yang [5] have studied the effect of thermal and electrical wall conductances on the same problem.

The above studies, however, do not take into account heat transfer by radiation, which will be significant when we are concerned with space technology and higher operating temperatures. The non-magnetic fully-developed, laminar convection flow in a vertical channel in the optically thin limit has been studied by Greif *et al.* [6] whereas Gupta and Gupta [7] studied the same problem in the presence of a transverse magnetic field.

In the present paper, we have studied the effect of radiation on the combined free and forced convection flow of an electrically conducting fluid between two arbitrary thermally and electrically conducting vertical parallel walls having a constant vertical temperature gradient. An exact solution of the governing equations has been obtained. The effect of the dimensionless physical parameters characterizing the flow on the velocity, the magnetic field, the temperature, the flow rate and the rate of heat transfer have been studied in detail.

GOVERNING EQUATIONS AND THEIR SOLUTION

Consider the combined free and forced convection between two parallel vertical walls, due to a constant pressure gradient in the presence of a uniform transverse magnetic field H_0. We employ a cartesian coordinate system with origin at the central line of the channel, z-axis along the vertical direction and y-axis along the direction of the magnetic field.

For a fully-developed laminar flow, the velocity and the magnetic field are given by $[0, 0, u(y)]$ and $[0, H_0, H_z(y)]$ respectively.

Assuming that the wall temperature has a uniform gradient N along the z-direction the temperature of the fluid can be assumed as

$$T = T^*(y) + Nz. \tag{1}$$

The y- and z-components of the momentum equations for the fully-developed steady flow are

$$\frac{\partial p}{\partial y} + \mu_e H_0 \frac{dH_z}{dy} = 0, \tag{2}$$

$$\nu \frac{d^2u}{dy^2} + \frac{\mu_e H_0}{\rho}\frac{dH_z}{dy} + g\beta(\theta^* + Nz) - \frac{1}{\rho}\frac{\partial p}{\partial z} = 0, \tag{3}$$

where $\theta^* = T^* - T_{w0}$.

The energy and the magnetic induction equations are

$$Nu = \alpha \frac{d^2\theta^*}{dy^2} - \frac{1}{\rho c_p}\frac{\partial q_R}{\partial y}, \tag{4}$$

$$\frac{d^2H_z}{dy^2} + \sigma\mu_e H_0 \frac{du}{dy} = 0. \tag{5}$$

In the energy equation we have neglected viscous and Joulean dissipation but included the heat radiation emitted by the boundaries. The fluid does not absorb its own emitted radiation in the optically thin limit. In other words, there is no self-absorption but the fluid does absorb radiation emitted by the boundaries. It has been shown by Cogley *et al.* [8] that in the optically thin limit for a non-gray gas near equilibrium, the following relation holds

$$\frac{\partial q_R}{\partial y} = 4(T - T_w)\int_0^\infty k_{\lambda w}\left(\frac{de_{b\lambda}}{dT}\right)_w d\lambda. \tag{6}$$

Using (6), equation (4) becomes

$$Nu = \alpha \frac{d^2\theta^*}{dy^2} - c\theta^*, \tag{7}$$

where

$$c = \frac{4}{\rho c_p}\int_0^\infty k_{\lambda 0}\left(\frac{de_{b\lambda}}{dT}\right)_0 d\lambda. \tag{8}$$

Subscript "0" indicates that all quantities have been evaluated at the reference temperature T_{w0} which is the temperature of the wall at $z = 0$. Hence our study will be limited to small difference of wall temperature to the fluid temperature.

Integrating equation (2) we get

$$p = -\frac{\mu_e H_z^2}{2} + f(z). \tag{9}$$

Using equation (9) in (3) we have

$$\nu \frac{d^2u}{dy^2} + \frac{\mu_e H_0}{\rho}\frac{dH_z}{dy} + g\beta\theta^* = \frac{1}{\rho}\frac{\partial f}{\partial z} - g\beta Nz. \tag{10}$$

Since u and H_z are functions of y only both sides of equation (10) must be equal to a constant c_1 (say).

Thus, we rewrite equation (10) as

$$\nu \frac{d^2u}{dy^2} + \frac{\mu_e H_0}{\rho}\frac{dH_z}{dy} + g\beta\theta^* = -c_1, \tag{11}$$

where

$$c_1 = -\left[\frac{1}{\rho}\frac{\partial f}{\partial z} - g\beta Nz\right]. \tag{12}$$

Introducing the following dimensionless variables

$$\eta = y/L, \quad u_1 = uL/\alpha c_2, \quad h = H_z/\sigma\mu_e \alpha c_2 H_0, \quad \theta = -\theta^*/NLc_2, \quad c_2 = c_1 L^3/\nu\alpha, \tag{13}$$

the equations (11), (7) and (5) become

$$\frac{d^2u_1}{d\eta^2} + M^2\frac{dh}{d\eta} - Ra\theta = -1, \tag{14}$$

$$\frac{d^2\theta}{d\eta^2} - F\theta = -u_1, \tag{15}$$

$$\frac{d^2h}{d\eta^2} + \frac{du_1}{d\eta} = 0, \tag{16}$$

where M, F and Ra are the dimensionless parameters defined in the nomenclature.

The boundary conditions on u_1 are

$$u_1 = 0 \quad \text{at} \quad \eta = \pm 1. \tag{17}$$

Assuming σ_w as the electrical conductivity and d as the thickness of the walls, the boundary conditions on h are given by Shercliff [9] as

$$\frac{dh}{d\eta} \pm \frac{h}{\phi} = 0 \quad \text{at} \quad \eta = \pm 1. \tag{18}$$

The boundary conditions on temperature due to Yu and Yang [5] are given by

$$\frac{d\theta}{d\eta} \pm \frac{\theta}{\psi} = 0 \quad \text{at} \quad \eta = \pm 1. \tag{19}$$

Eliminating u_1 and h from (14), (15) and (16) we get

$$\frac{d^4\theta}{d\eta^4} - (F + M^2)\frac{d^2\theta}{d\eta^2} + (M^2F + Ra)\theta = c_3, \tag{20}$$

where c_3 is a constant and its solution can be readily determined. The solution of u_1 and h can then be obtained. The solution for θ, u_1 and h satisfying the boundary conditions (17)–(19) are

$$\theta = a_8[1 - a_3\cosh m_1\eta - (a_2 - a_1a_3)\cosh m_2\eta], \tag{21}$$

$$u_1 = a_8[F - a_3(F - m_1^2)\cosh m_1\eta - (a_2 - a_1a_3)(F - m_2^2)\cosh m_2\eta], \tag{22}$$

$$h = a_8\left[Ra\eta + a_3a_4\frac{\sinh m_1\eta}{m_1} + (a_2 - a_1a_3)a_5\frac{\sinh m_2\eta}{m_2}\right], \tag{23}$$

where

$$a_1 = \frac{\psi m_1\sinh m_1 + \cosh m_1}{\psi m_2\sinh m_2 + \cosh m_2},$$

$$a_2 = \frac{1}{\psi m_2\sinh m_2 + \cosh m_2},$$

$$a_3 = \frac{F - a_2(F - m_2^2)\cosh m_2}{(F - m_1^2)\cosh m_1 - a_1(F - m_2^2)\cosh m_2},$$

$$a_4 = m_1^2(F - m_1^2) - Ra, \quad a_5 = m_2^2(F - m_2^2) - Ra,$$

$$a_6 = Ra + a_3 a_4 \cosh m_1 + (a_2 - a_1 a_3) a_5 \cosh m_2, \quad (24)$$

$$a_7 = Ra + a_3 a_4 \frac{\sinh m_1}{m_1} + (a_2 - a_1 a_3) a_5 \frac{\sinh m_2}{m_2},$$

$$a_8 = (1+\phi)/(\phi a_6 + a_7), \quad c_3 = a_8(M^2 F + Ra),$$

$$m_1^2 = \tfrac{1}{2}[M^2 + F + \sqrt{\{(F - M^2)^2 - 4Ra\}}],$$

$$m_2^2 = \tfrac{1}{2}[M^2 + F - \sqrt{\{(F - M^2)^2 - 4Ra\}}].$$

The velocity, the magnetic field and the temperature are dependent on the parameters M, F, Ra, ϕ and ψ and the effect of the wall conductances can be studied with the help of the parameters ϕ and ψ.

RESULTS AND DISCUSSIONS

In order to study the effects of wall conductances on the hydromagnetic free and forced convective flow of the ratiating gas, we have presented the velocity, the magnetic field and the temperature in Figs. 1–3 for various values of ϕ and ψ on taking $M^2 = 10$, $F = 2$ and $Ra = 1$. It is found from Figs. 1–3 that for fixed values of ϕ, the velocity and the magnetic field decrease while the temperature increases with the increase of ψ.

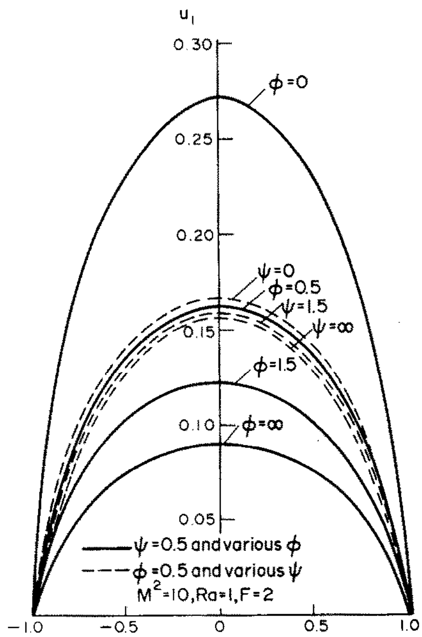

FIG. 1. Velocity profiles against η.

For fixed ψ, it is also clear from the same figures that the velocity and the temperature decrease whereas the magnetic field increases with increase of ϕ.

The non-dimensional flow rate

$$w\left(= \int_{-1}^{1} u_1 \, d\eta\right)$$

and the rate of heat transfer

$$G\left[= \left(-\frac{d\theta}{d\eta}\right)_{\eta=1}\right]$$

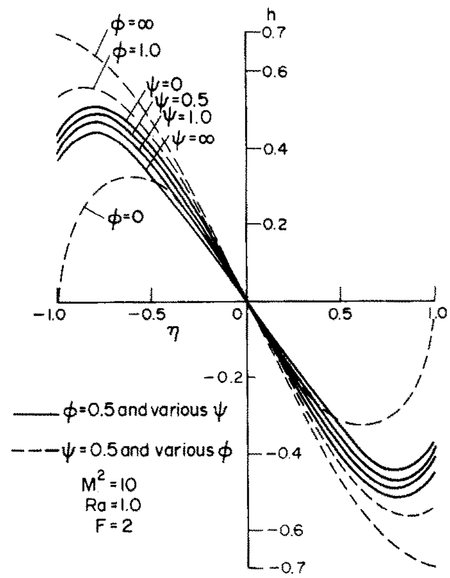

FIG. 2. Magnetic field against η.

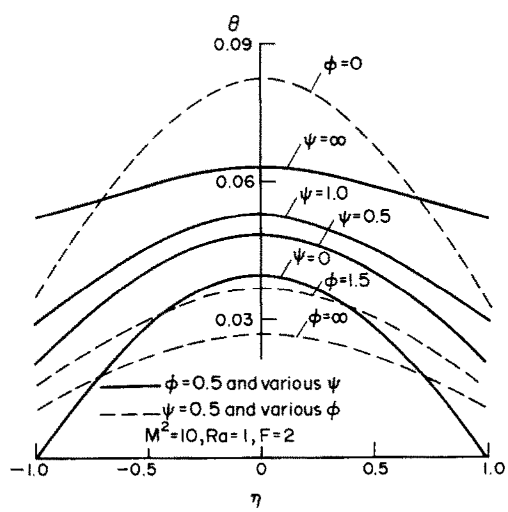

FIG. 3. Temperature profiles against η.

are given by

$$w = 2a_8\left[F - (a_2 - a_1 a_3)(F - m_2^2)\frac{\sinh m_2}{m_2} - a_3(F - m_1^2)\frac{\sinh m_1}{m_1}\right], \quad (25)$$

$$G = a_8[a_3 m_1 \sinh m_1 + (a_2 - a_1 a_3) m_2 \sinh m_2]. \quad (26)$$

In the absence of radiation ($F = 0$), equations (22), (21), (23), (25) and (26) are identical with equations (12), (13), (14), (17) and (18) respectively of Yu and Yang [5].

In the absence of wall conductances ($\phi = 0$, $\psi = 0$) equations (21)–(23), (25) and (26) reduce to

$$\theta = \frac{1}{a^*}\left[1 - \frac{m_2^2}{m_2^2 - m_1^2}\frac{\cosh m_1 \eta}{\cosh m_1} + \frac{m_1^2}{m_2^2 - m_1^2}\frac{\cosh m_2 \eta}{\cosh m_2}\right], \quad (27)$$

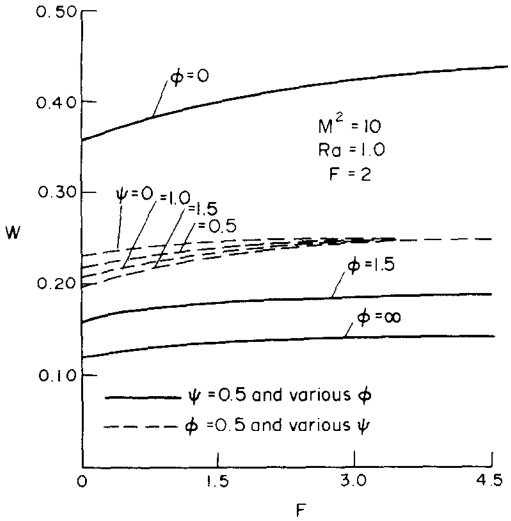

FIG. 4. Non-dimensional flow rate vs F.

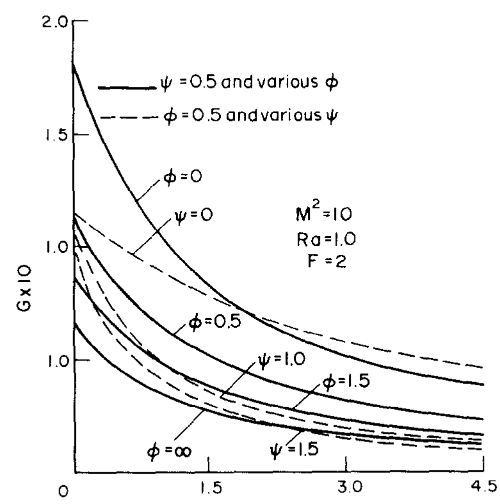

FIG. 5. Non-dimensional rate of heat transfer vs F.

$$u_1 = \frac{1}{a^*}\left[F - \frac{m_2^2(F-m_1^2)}{m_2^2-m_1^2}\frac{\cosh m_1\eta}{\cosh m_1} + \frac{m_1^2(F-m_2^2)}{m_2^2-m_1^2}\frac{\cosh m_2\eta}{\cosh m_2}\right], \quad (28)$$

$$h = \frac{1}{a^*(m_2^2-m_1^2)}\left[\frac{m_2^2(F-m_1^2)}{m_1}\left\{\frac{\sinh m_1\eta}{\cosh m_1} - \eta\tanh m_1\right\} + \frac{m_1^2(F-m_2^2)}{m_2}\left\{\eta\tanh m_2 - \frac{\sinh m_2\eta}{\cosh m_2}\right\}\right], \quad (29)$$

$$w = \frac{2}{a^*(m_2^2-m_1^2)}\left[F(m_2^2-m_1^2) - \frac{m_2^2(F-m_1^2)}{m_1}\tanh m_1 + \frac{m_1^2(F-m_2^2)}{m_2}\tanh m_2\right], \quad (30)$$

$$G = \frac{m_1^2 m_2^2}{a^*(m_2^2-m_1^2)}\left[\frac{\tanh m_1}{m_1} - \frac{\tanh m_2}{m_2}\right], \quad (31)$$

where

$$a^* = Ra + \frac{M^2m_2^2(F-m_1^2)}{m_2^2-m_1^2}\frac{\tanh m_1}{m_1} - \frac{M^2m_1^2(F-m_2^2)}{m_2^2-m_1^2}\frac{\tanh m_2}{m_2}. \quad (32)$$

Equations (27)–(31) are identical with equations (20)–(22), (24) and (25) of Gupta and Gupta [7], provided

$$\frac{c_4}{M^2F+Ra} = \frac{1}{a^*}. \quad (33)$$

Gupta and Gupta [7] left $c_4(=M^2c_3-c_2)$ as unknown. In our case c_4 is not present and evaluation of it has been made on considering the fact that total current flowing between the walls of the channel is zero.

In Figs. 4 and 5 the non-dimensional flow rate w and rate of heat transfer G have been plotted against F for $M^2 = 10$, $Ra = 1$ and for various values of ϕ and ψ. It is observed from Figs. 4 and 5 that for fixed F and ψ, the flow rate and the rate of heat transfer decrease with the increase of ϕ. It is also observed that for fixed F and ϕ, w and G decrease with increase of ψ. But for fixed ϕ and ψ, w increases while G decreases with the increase of F. It is seen from Figs. 4 and 5 that due to presence of radiation ($F \neq 0$), volume flow is more while rate of heat transfer is less than those in the absence of radiation ($F = 0$).

It is interesting to note that the effects of ϕ and ψ are similar to those of M and Ra respectively on the velocity, the magnetic field, the temperature, the flow rate and the rate of heat transfer.

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EFFET DES CONDUCTANCES DE LA PAROI SUR LA CONVECTION HYDROMAGNETIQUE D'UN GAZ RAYONNANT DANS UN CANAL VERTICAL

Résumé—On donne une solution exacte de la convection établie d'un fluide électriquement conducteur s'écoulant entre deux parois verticales conductrices de la chaleur et de l'électricité et soumis à un champ magnétique transversal. Il apparait qu'une augmentation de la conductance électrique des parois produit une diminution de la vitesse, de la température, du débit de fluide et du flux thermique et une augmentation du champ magnétique. Cependant, une augmentation de la conductance thermique des parois produit une diminution de la vitesse, du champ magnétique, du débit de fluide et du flux thermique et une augmentation de la température.

DER EINFLUSS DES WÄRMELEITVERMÖGENS DER WAND AUF DIE HYDROMAGNETISCHE KONVEKTION EINES STRAHLENDEN GASES IN EINEM VERTIKALEN KANAL

Zusammenfassung—Es wurde eine exakte Lösung für die voll ausgebildete konvektive Strömung eines elektrisch leitenden Fluides zwischen zwei thermisch und elektrisch leitenden vertikalen Wänden unter der Einwirkung eines transversalen Magnetfeldes gefunden. Es wurde festgestellt, daß die Geschwindigkeit, die Temperatur, der Massenstrom und der Wärmeübergang mit wachsender elektrischer Leitfähigkeit der Wände abnimmt, während das Magnetfeld zunimmt. Nimmt die Wärmeleitfähigkeit der Wände zu, so nimmt auch die Temperatur zu, und die Geschwindigkeit, das Magnetfeld, der Massenstrom und der Wärmeübergang nehmen ab.

ВЛИЯНИЕ ТЕПЛОПРОВОДНОСТИ СТЕНОК НА ГИДРОМАГНИТНУЮ КОНВЕКЦИЮ ИЗЛУЧАЮЩЕГО ГАЗА В ВЕРТИКАЛЬНОМ КАНАЛЕ

Аннотация — Получено точное решение для полностью развитого конвективного течения электрически проводящей жидкости между двумя вертикальными термически и электрически проводящими стенками под действием поперечного магнитного поля. Найдено, что с увеличением электропроводности стенок уменьшаются скорость, температура, расход жидкости и интенсивность теплообмена, а величина магнитного поля возрастает. В то время как уменьшаются величина магнитного поля, расход жидкости и интенсивность теплообмена, температура повышается с увеличением теплопроводности стенок.